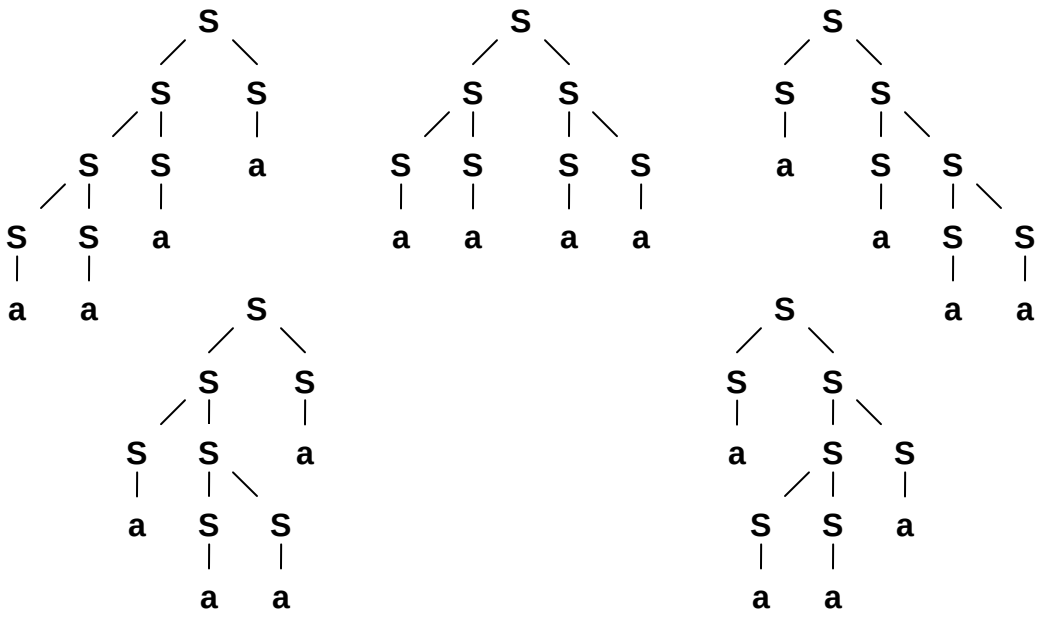
CS 603 Exam 1 **Spring 2006** **Solution**

Problem 7 is 15 points. Each other problem is 10 points. Maximum possible is 105/100.

1. Draw all possible parse trees for the string aaaa using the grammar $S \rightarrow SS \mid a$.



2. Write an unambiguous CFG or BNF grammar (<u>not</u> EBNF) for arithmetic expressions with binary operators @, #, $, and &. Operator @ is left-associative and has highest precedence. Operator # is right-associative and has second highest precedence. Operator $ is left-associative and has third highest precedence. Operator & is right-associative and has lowest precedence. All the operands can be denoted by id. Handling parentheses is not necessary. Here is an example input string: id @ id @ id # id # id $ id $ id & id & id.

E → X & E | X
X → X $ Y | Y
Y → Z # Y | Z
Z → Z @ *id* | *id*

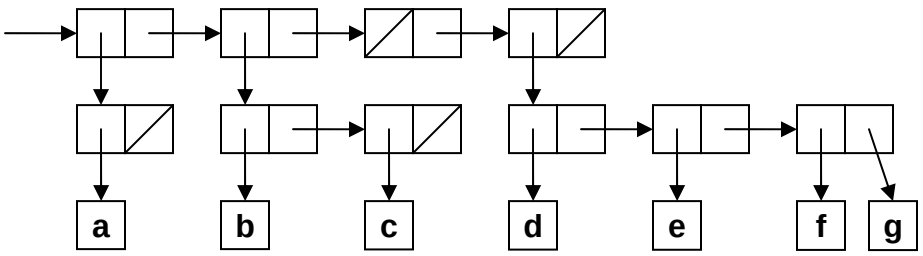
3. Write a recursive Impcore function (binary_to_decimal *m*) that converts *m* from binary to decimal. You may use helper functions if you wish, but do not use while or set expressions. Example: (binary_to_decimal 1100) should return 12.

```
(define binary_to_decimal (m)
   (if (= m 0) 0
       (+ (* 2 (binary_to_decimal (/ m 10))) (mod m 10))))
```

4. Complete this natural operational semantics rule for a C-like DO_WHILE(e_1, e_2) in Impcore The expression is evaluated for its side-effects, so the return value is unimportant. Here is an example as it could appear in a language with an Impcore-like syntax: (do_while (set x (* 10 x)) (x < 1000)). The expression e_1 is the body of the loop, and the expression e_2 is the condition. Expression e_1 must be evaluated before e_2. [Hints: Think about how to define DO_WHILE in terms of WHILE. Two premises should be sufficient.]

$$\frac{\langle e_1, \xi, \phi, \rho\rangle \Downarrow \langle \mathbf{v_1}, \xi', \phi, \rho'\rangle \quad \langle \text{WHILE}(\mathbf{e_2}, \mathbf{e_1}), \xi', \phi, \rho'\rangle \Downarrow \langle \mathbf{v_2}, \xi'', \phi, \rho''\rangle}{\langle \text{DO\_WHILE}(e_1, e_2), \xi, \phi, \rho\rangle \Downarrow \langle \mathbf{v_2}, \xi'', \phi, \rho''\rangle} \quad \text{(DO\_WHILE)}$$

5. Draw a diagram that illustrates the internal representation for this S-expression: ((a) (b c) ( ) (d e f . g))



6. Write a Scheme function (inner X Y f g) that returns the inner product of the lists X and Y with respect to binary operations f and g. You may use helper functions if you wish, but do not use while or set expressions. You may assume that lists X and Y are nonempty lists of the same length, and that operation f is associative. Example: (inner '(1 2 3) '(4 5 6) + *) should return 32, because 1*4+2*5+3*6 = 32. This kind of inner product is sometimes also called a dot product. Another example: (inner '(1 2 3) '(4 5 6) * +) should return 315, because (1+4)*(2+5)*(3+6) = 315.

```
(define inner (X Y f g)
    (if (null? (cdr X)) (g (car X) (car Y))
        (f (g (car X) (car Y)) (inner (cdr X) (cdr Y) f g))))
```

7. Write a Scheme function (outer X Y f) that returns the outer product of the lists X and Y with respect to binary operation f. You may use helper functions if you wish, but do not use while or set expressions. The result is a list of length |X|, each of whose members is a sublist of length |Y|. The k^{th} element of the j^{th} sublist is obtained by applying operation f to the j^{th} element of X and the k^{th} element of Y. Example: (outer '(1 2 3) '(4 5 6 7) *) should return the list ((4 5 6 7) (8 10 12 14) (12 15 18 21)). This kind of outer product is sometimes also called a multiplication table. [Hints: Write a helper function that constructs one of the sublists. Also, consider using the map function.]

```
(define helper (a Y f)
    (map ((curry f) a) Y))

(define outer (X Y f)
    (map (lambda (a) (helper a Y f)) X))
```

OR

```
(define helper (a Y f )
    (if (null? Y) '( )
        (cons (f a (car Y)) (helper a (cdr Y) f))))

 (define outer (X Y f)
    (if (null? X) '( )
        (cons (helper (car X) Y f) (outer (cdr X) Y f))))
```

8. Write the output that would be generated by the μScheme interpreter after entering each line of code below.

```
(val m (lambda (w) (lambda (op x) (set w (op w x)))))        ; m
(val a (m 10))     ; procedure
(val b (m 20))     ; procedure
(val c (m 50))     ; procedure
(a * 3)     ; 30
(b + 5)     ; 25
(c + 4)     ; 54
(a + 6)     ; 36
(b * 2)     ; 50
(c – 10)    ; 44
(a – 1)     ; 35
(b – 2)     ; 48
(c / 2)     ; 22
```

9. Consider this Scheme expression: (f (* a b) (* a b) (* a b))

   a. Using let, write an equivalent expression that only evaluates (* a b) once.

   (let ((z (* a b))) (f z z z))

   b. Using lambda, write an equivalent expression that only evaluates (* a b) once.

   ((lambda (z) (f z z z)) (* a b))

10. Complete these natural semantics rules for Scheme's primitive car and cdr functions.

$$\frac{\langle e, \rho, \sigma_0\rangle \Downarrow \langle \text{PRIMITIVE}(\texttt{car}), \sigma_1\rangle \quad \langle e_1, \rho, \sigma_1\rangle \Downarrow \langle \textbf{CONS}(\ell_1, \ell_2), \sigma_2\rangle}{\langle \text{APPLY}(e, e_1), \rho, \sigma_0\rangle \Downarrow \langle \sigma_2(\ell_1), \sigma_2\rangle} \quad \text{(CAR)}$$

$$\frac{\langle e, \rho, \sigma_0\rangle \Downarrow \langle \textbf{PRIMITIVE}(\texttt{cdr}), \sigma_1\rangle \quad \langle e_1, \rho, \sigma_1\rangle \Downarrow \langle \textbf{CONS}(\ell_1, \ell_2), \sigma_2\rangle}{\langle \text{APPLY}(e, e_1), \rho, \sigma_0\rangle \Downarrow \langle \sigma_2(\ell_2), \sigma_2\rangle} \quad \text{(CDR)}$$