

1. For each ML function definition below, write the type (usually a polymorphic type) that the ML interpreter would infer for the function. [20 points]

a. `fun f _ = [];`

fn : _____

b. `fun f (x,y) = (y,x);`

fn : _____

c. `fun f (x,y,z) = [x,y,z];`

fn : _____

d. `fun f (w,x,y,z) = (w div x, y/z);`

fn : _____

e. `fun f x y = let val g = op^ in g(x,y) end;`

fn : _____

f. `fun f (x,y,z) = z (y x);`

fn : _____

g. `fun f x y z = x y z;`

fn : _____

h. `fun f w x y z = if w then x else y=z;`

fn : _____

i. `fun f (w,x) = let fun g (y,z) = y (y z) in g (w,x) end;`

fn : _____

j. `fun f (u,[]) = [u] | f([],v) = [v] | f(w::x,y::z) = [w,y]::f(x,z);`

fn : _____

2. Write an ML function `inner(X,Y,f,g)` that returns the inner product of the lists `X` and `Y` with respect to binary operations `f` and `g`. Assume that lists `X` and `Y` are nonempty lists of the same length, and that operation `f` is associative. You may use helper functions if you wish. For maximum credit, do not call the predefined `hd` or `tl` functions. Example: `inner([1,2,3], [4,5,6], op+, op*)` should return `32`, because $1*4+2*5+3*6 = 32$. This kind of inner product is often called a dot product. Another example: `inner([1,2,3], [4,5,6], op*, op+)` should return `315`, because $(1+4)*(2+5)*(3+6) = 315$. **[10 points]**
3. Write an ML function `outer(X,Y,f)` that returns the outer product of the lists `X` and `Y` with respect to binary operation `f`. You may use helper functions if you wish. For maximum credit, do not call the predefined `hd` or `tl` functions. The result is a list of length $|X|$, each of whose members is a sublist of length $|Y|$. The k^{th} element of the j^{th} sublist is obtained by applying operation `f` to the j^{th} element of `X` and the k^{th} element of `Y`. Example: `outer([1,2,3], [4,5,6,7], op*)` should return the list `[[4,5,6,7],[8,10,12,14],[12,15,18,21]]`. This kind of outer product is often called a multiplication table. Hints: Write a helper function that constructs one of the sublists. Also, consider using the `map` function. **[10 points]**

7. First read this μ Smalltalk code. Next determine the values that would be obtained upon sending each of the messages {p, q, r, s, t} to each of the receiver objects {b, c, d, e, f, g}. Write these values in the table below, where each row corresponds to a message and each column corresponds to a receiver. **[15 points]**

```
(class A Object ()
  (method p () (q self))
  (method q () (r self))
  (method r () (s self))
  (method s () (t self))
  (method t () 1)
)
(class B A ()
  (method p () (s self))
  (method r () 2)
)
(class C B ()
  (method q () (p self))
  (method s () 3)
)
(class D C ()
  (method p () 4)
  (method t () (r self))
)
(class E D ()
  (method q () 5)
  (method s () (t self))
)
(class F E ()
  (method r () (q self))
  (method t () 6)
)
(class G F ()
  (method p () (q self))
  (method r () (t self))
  (method s () 7))

(val b (new B))
(val c (new C))
(val d (new D))
(val e (new E))
(val f (new F))
(val g (new G))
```

	b	c	d	e	f	g
p						
q						
r						
s						
t						

8. Write a μ Smalltalk class `Queue` such that each message send in the client would produce the output or return value as indicated below. [20 points]

```
(class Node Object (data next)
  (method data ( ) data)
  (method next ( ) next)
  (method data: (x) (set data x))
  (method next: (x) (set next x))
)
; definition of class Queue should go here
```

```
(val Q (new Queue))
(isEmpty Q)           ; <True>
(enqueue: Q #a)       ; a
(enqueue: Q #b)       ; b
(enqueue: Q #c)       ; c
(isEmpty Q)           ; <False>
(display Q)           ; a b c
(dequeue Q)           ; a
(dequeue Q)           ; b
(dequeue Q)           ; c
(isEmpty Q)           ; <True>
(dequeue Q)           ; nil
```