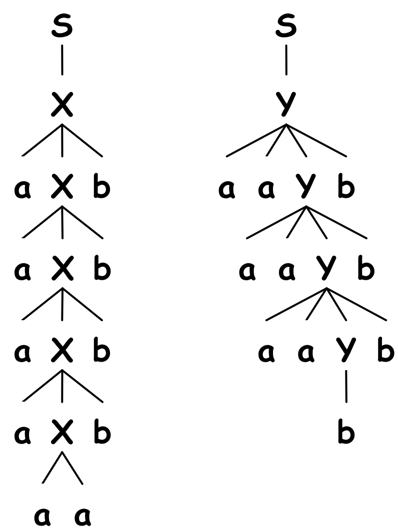
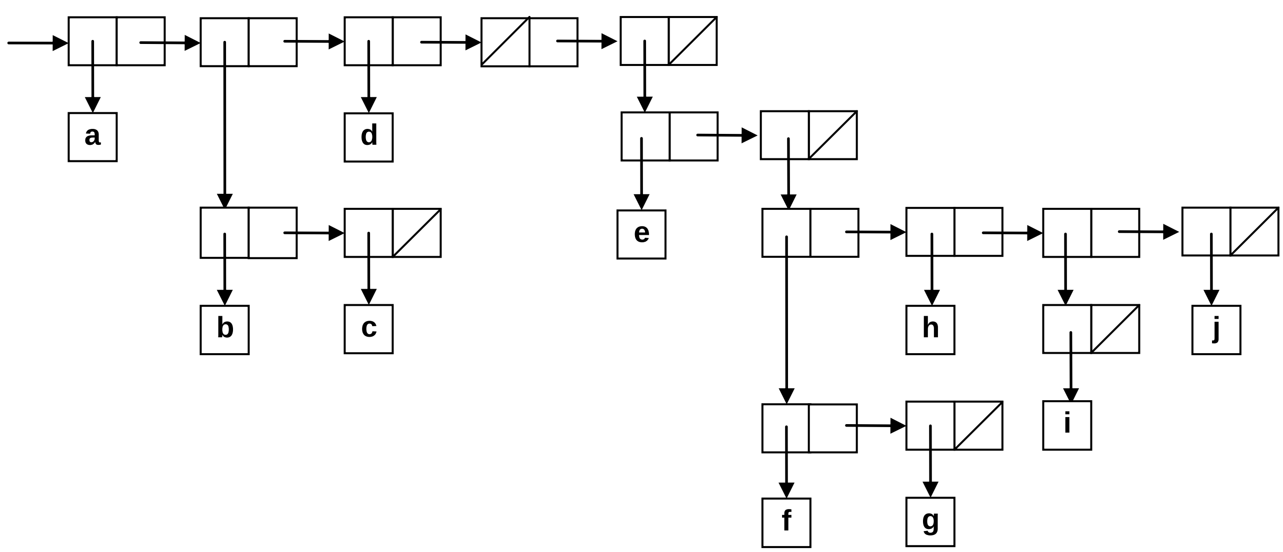
CS 603 Exam 1 **Spring 2008** **Solution**

1. Show that this grammar is ambiguous by drawing two parse trees that generate the same string.

   $S \rightarrow X \mid Y$
   $X \rightarrow aXb \mid aa$
   $Y \rightarrow aaYb \mid b$



2. Draw a diagram that illustrates the internal representation of this S-expression:
   (a (b c) d ( ) (e ((f g) h (i) j)))



3. Write an unambiguous context-free grammar that generates S-expressions that contain only the symbol x. For example, (x (x x) x ( ) (x ((x x) x (x) x))). Your production rules may use concatenation and |, but not other extended BNF operations.

S → (L) | x
L → SL | ε

4. Suppose we give Impcore a new primitive function && which implements binary short-circuit AND. So the expression (&& x y) in Impcore should be equivalent to x&&y in C or C++ or Java. Define the && function by completing these two Impcore-style natural operational semantics rules.

$\phi(f) = \text{PRIMITIVE}(\&\&)$ $\langle \mathbf{e_1}, \xi, \phi, \rho \rangle \Downarrow \langle \mathbf{v_1}, \xi', \phi, \rho' \rangle$ $\mathbf{v_1 \neq 0}$ $\langle \mathbf{e_2}, \xi', \phi, \rho' \rangle \Downarrow \langle \mathbf{v_2}, \xi'', \phi, \rho'' \rangle$ ――――――――――――― $\langle \text{APPLY}(f,e_1,e_2), \xi, \phi, \rho \rangle \Downarrow \langle \mathbf{v_2}, \xi'', \phi, \rho'' \rangle$	$\phi(f) = \text{PRIMITIVE}(\&\&)$ $\langle \mathbf{e_1}, \xi, \phi, \rho \rangle \Downarrow \langle \mathbf{v_1}, \xi', \phi, \rho' \rangle$ $\mathbf{v_1 = 0}$ ――――――――――――― $\langle \text{APPLY}(f,e_1,e_2), \xi, \phi, \rho \rangle \Downarrow \langle \mathbf{0}, \xi', \phi, \rho' \rangle$

5. Suppose we give μScheme a new primitive function || which implements binary short-circuit OR. So the expression (|| x y) in μScheme should be equivalent to x||y in C or C++ or Java. Define the || function by completing these two μScheme-style natural operational semantics rules.

$\langle e, \rho, \sigma \rangle \Downarrow \langle \text{PRIMITIVE}(\|\|), \sigma_1 \rangle$ $\langle \mathbf{e_1}, \rho, \sigma_1 \rangle \Downarrow \langle \mathbf{v_1}, \sigma_2 \rangle$ $\mathbf{v_1 \neq BOOL(\#f)}$ ――――――――――――― $\langle \text{APPLY}(e,e_1,e_2), \rho, \sigma \rangle \Downarrow \langle \mathbf{v_1}, \sigma_2 \rangle$	$\langle e, \rho, \sigma \rangle \Downarrow \langle \text{PRIMITIVE}(\|\|), \sigma_1 \rangle$ $\langle \mathbf{e_1}, \rho, \sigma_1 \rangle \Downarrow \langle \mathbf{v_1}, \sigma_2 \rangle$ $\mathbf{v_1 = BOOL(\#f)}$ $\langle \mathbf{e_2}, \rho, \sigma_2 \rangle \Downarrow \langle \mathbf{v_2}, \sigma_3 \rangle$ ――――――――――――― $\langle \text{APPLY}(e,e_1,e_2), \rho, \sigma \rangle \Downarrow \langle \mathbf{v_2}, \sigma_3 \rangle$

6. Write a μScheme function (diagonal M) where M is a square matrix stored as a list of row lists. It should return a list of the main diagonal elements of M. Example: (diagonal '((a b c d) (e f g h) (i j k l) (m n o p))) returns (a f k p).

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

```
(define diagonal (M) (if (null? M) '( )
      (cons (car (car M)) (diagonal (map cdr (cdr M))))))
```

7. Write a μScheme function (scan op id L) where op is a binary function, id is the identity value, and L is a list. It should return a list of the values obtained by folding op across each possible prefix of L. Example: (scan + 0 '(2 3 5 7 11)) returns (0 2 2+3 2+3+5 2+3+5+7 2+3+5+7+11) = (0 2 5 10 17 28). [You may assume that op is an associative operation.]

```
(define scan (op id L) (if (null? L) (cons id '( ))
      (cons id (map ((curry op) (car L)) (scan op id (cdr L))))))
```

8. The Impcore function below shows an inefficient way to compute Fibonacci numbers. Note that (fib 0) = (fib 1) = 1, (fib 2) = 2, (fib 3) = 3, (fib 4) = 5, (fib 5) = 8, etc. Write an equivalent function so that (fib n) runs in O(n) time. Hint: use a helper function that "remembers" the previous two values.

```
(define fib (n) (if (<= n 1) 1
      (+ (fib (- n 2)) (fib (- n 1)))))
```

```
(define fib (n) (helper n 1 1))

(define helper (n x y) (if (<= n 1) y
      (helper (- n 1) y (+ x y))))
```

9. Complete the μScheme function Stack so that the print statements in the client code below will produce the given output.

(define Stack ( ) (let ((L '( ))) (lambda (m) **(if (= m 'isEmpty) (null? L)** **(if (= m 'top) (car L)** **(if (= m 'pop) (set L (cdr L))** **(if (= m 'push)** **(lambda (v) (set L (cons v L)))** **'error** **))))** **)))**	(val A (Stack)) (val B (Stack)) (val k 0) (while (< k 10) (begin ((A 'push) k) ((B 'push) (+ k 1)) (set k (+ k 2)))) (while (not (A 'isEmpty)) (begin (print (A 'top)) (A 'pop))) (while (not (B 'isEmpty)) (begin (print (B 'top)) (B 'pop)))	8 6 4 2 0 9 7 5 3 1

10. Using either C or C++ or Java, write a definition for a Stack abstract data type represented as a linked list of ints. Provide these operations with the same functionality as in the preceding problem: isEmpty, top, pop, push.

```
// Java
class Node {
	int value;
	Node next;
	Node(int v, Node n) { value=v; next=n; }
}
class Stack {
	Node head;
	Stack( ) { head = null; }
	boolean isEmpty( ) { return head==null; }
	int top( ) { return head.value; }
	void pop( ) { head = head.next; }
	void push(int v) { head = new Node(v, head); }
}
```